Matrix algebra – addition and multiplication

Exercise 2.6

Determine for what values of u and v the following equality holds:

$$\begin{pmatrix} (1-u)^2 & v^2 & 3 \\ v & 2u & 5 \\ 6 & u & -1 \end{pmatrix} = \begin{pmatrix} 4 & 4 & u \\ v & -3v & u-v \\ 6 & v+5 & -1 \end{pmatrix}$$

Solution Exercise 2.6

$u = 3$

$u - v = 5$

$v = -2$

Exercise 2.7

Evaluate $A + B,\ \ A - B$ and $5A - 3B$ when

$$A = \begin{pmatrix} 0 & 1 & -1 \\ 2 & 3 & 7 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & -1 & 5 \\ 0 & 1 & 9 \end{pmatrix}$$

Solution Exercise 2.7

$$A + B = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 4 & 16 \end{pmatrix}$$

$$A - B = \begin{pmatrix} -1 & 2 & -6 \\ 2 & 2 & -2 \end{pmatrix}$$

$$5A - 3B = \begin{pmatrix} -3 & 8 & -20 \\ 10 & 12 & 8 \end{pmatrix}$$

Exercise 2.8

a. If $x + 0 = 0$, what do you know about the components of x
b. If $0x = 0$, what do you know about the components of x?
c. Solve the vector equation $4x - 7a = 2x + 8b - a$ for x in terms of vectors a and b.

Solution Exercise 2.8

a. All components of x are 0 if $x + 0 = 0$.

b. Nothing is known about the components of x if $0x = x$.

c. $$\begin{aligned} 4x - 7a &= 2x + 8b - a \\ 2x &= 8b + 6a \\ x &= 4b + 3a \end{aligned}$$

Exercise 2.9

Compute the products AB and BA, if possible, when A and B are, respectively

a. $A = \begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 4 \\ 1 & 5 \end{pmatrix}$

b. $A = \begin{pmatrix} 8 & 3 & -2 \\ 1 & 0 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -2 \\ 4 & 3 \\ 1 & -5 \end{pmatrix}$

c. $A = \begin{pmatrix} -1 & 0 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}$

d. $A = \begin{pmatrix} 0 \\ -2 \\ 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -2 & 3 \end{pmatrix}$

Solution Exercise 2.9

a. $AB = \begin{pmatrix} -2 & -10 \\ -2 & 17 \end{pmatrix}$ and $BA = \begin{pmatrix} 12 & 6 \\ 15 & 3 \end{pmatrix}$

b. $AB = \begin{pmatrix} 26 & 3 \\ 6 & -22 \end{pmatrix}$ and $BA = \begin{pmatrix} 14 & 6 & -12 \\ 35 & 12 & 4 \\ 3 & 3 & -22 \end{pmatrix}$

c. $AB = \text{"cannot be determined"}$ and $BA = \begin{pmatrix} -1 & 4 \\ 3 & 4 \\ 4 & 8 \end{pmatrix}$

d. $AB = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 & -6 \\ 0 & -8 & 12 \end{pmatrix}$ and $BA = 16$

Exercise 2.10

Show that $A(BC) = (AB)C$ with:

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}, B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}, C = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix},$$

Solution Exercise 2.10

$$BC = \begin{pmatrix} 14 & -4 & 10 \\ 21 & 0 & 27 \\ 11 & -4 & 13 \end{pmatrix}$$

$$A(BC) = \begin{pmatrix} 23 & 8 & 25 \\ 92 & -28 & 76 \\ 4 & -8 & -4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 5 & 3 & 3 \\ 19 & -5 & 16 \\ 1 & -3 & 0 \end{pmatrix}$$

$$(AB)C = \begin{pmatrix} 23 & 8 & 25 \\ 92 & -28 & 76 \\ 4 & -8 & -4 \end{pmatrix}$$

$$A(BC) = (AB)C$$

Exercise 2.12

Find all matrices B that commute with:

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

in the sense that $AB = BA$

Solution Exercise 2.12

The general system of equations has to hold for A and B to commute

$$\begin{cases} a_{11}b_{11} + a_{12}b_{21} = b_{11}a_{11} + b_{12}a_{21} \\ a_{11}b_{12} + a_{12}b_{22} = b_{11}a_{12} + b_{12}a_{22} \\ a_{21}b_{11} + a_{22}b_{21} = b_{21}a_{11} + b_{22}a_{21} \\ a_{21}b_{12} + a_{22}b_{22} = b_{21}a_{12} + b_{22}a_{22} \end{cases}$$

$$\begin{cases} b_{11} + 2b_{21} = b_{11} + 2b_{12} \\ b_{12} + 2b_{22} = 2b_{11} + 3b_{12} \\ 2b_{11} + 3b_{21} = b_{21} + 2b_{22} \\ 2b_{12} + 3b_{22} = 2b_{21} + 3b_{22} \end{cases}$$

$$=$$

$$\begin{cases} b_{21} = b_{12} \\ b_{22} = b_{11} + b_{12} \\ b_{11} = -b_{21} + b_{22} \\ b_{12} = b_{21} \end{cases}$$

$$v = b_{11}, u = b_{12}$$

$$\Rightarrow B = \begin{pmatrix} v & u \\ u & v + u \end{pmatrix}$$